Management of Uncertainty in Medicine*

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ABSTRACT

This article discusses MUM, a knowledge-based consultation system designed to manage the uncertainty inherent in medical diagnosis. The primary task of the system is to plan which questions, tests, and treatments to order at each point in a consultation, given current uncertain knowledge about the patient's disease. Managing uncertainty means planning what to do when uncertain; the authors suggest that this ability must be designed in, not added on, to the architectures of knowledge-based systems. MUM is based on one such architecture, implemented as a generalized inference network and planner. The network facilitates local combination of evidence; the planner "reads" the state of the network after each piece of evidence integrated, then decides which evidence to seek on the basis of its several goals.

KEYWORDS: ***automated reasoning, diagnosis, planning, control, reasoning about uncertainty***

INTRODUCTION

MUM (Management of Uncertainty in Medicine) is a knowledge-based consultation system designed to manage the uncertainty inherent in medical

* This research is funded by a grant from the National Science Foundation, IST 8409623, and DARPA-RADC Contract, F30602-85-C-0014. The authors gratefully acknowledge the many contributions of Thomas Gruber to the MUM Project.

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International Journal of Approximate Reasoning 1987; 1:103–116

52 Vanderbilt Ave., New York, NY 10017 0888-613X/87/$3.50

diagnosis. Managing uncertainty means planning actions to minimize uncertainty or its consequences. Thus, it is a control problem—an issue for the component of a knowledge system that decides how to proceed from an uncertain state of a problem.

Uncertainty can be managed by many strategies, depending on the kind of problem one is trying to solve. These include asking for evidence, hedging one's bets, deciding arbitrarily and backtracking on failure, diversification or risk sharing, and worst-case analysis. The facility with which a consultation system such as MUM manages uncertainty is evident in the questions it asks: it should ask all necessary questions and no unnecessary questions, and it should ask its questions in the right order. These conditions, especially the last one, preclude uniform and inflexible control strategies. They prompted the development of the MUM architecture, in which control decisions are taken by reasoning about features of evidence and sources of uncertainty.

The Goals of MUM

MUM diagnoses diseases that manifest as chest pain and abdominal pain. This involves taking a history, asking for physical findings, ordering tests, and prescribing trial therapy. Physicians call a diagnostic sequence of questions and tests a workup. MUM's primary goal is to generate workups for chest and abdominal diseases that include, in the correct order, all necessary questions and tests and none that are superfluous. Because we built MUM to study the management of uncertainty, the goal of correct diagnosis is secondary to generating the correct workup. We were influenced by a distinction physicians make between *retrospective* diagnosis, in which all evidence is known in advance and the goal is to make a correct diagnosis, and *prospective* diagnosis, which emphasizes the workup and proper management of the patient, even under uncertainty about his or her condition. MUM is definitely prospective.

Figure 1 illustrates part of the workup for coronary artery disease. Clearly, we could build a system that follows this and other stored workups, but the point of the research is to be able to reason about the features of evidence, and the uncertainty in partially developed diagnoses, to decide which questions to ask next. If MUM does this properly, its questioning will correspond with a standard workup or at least be a reasonable alternative workup.

Managing Uncertainty and Control

MUM is based on the idea that managing uncertainty and controlling a complex knowledge system are manifestations of a single task, namely, acquiring evidence and using it to solve problems. There would be little basis for variation in problem-solving strategies if all evidence was equally costly, reliable, available, and pertinent; but if evidence is differentiated along these and

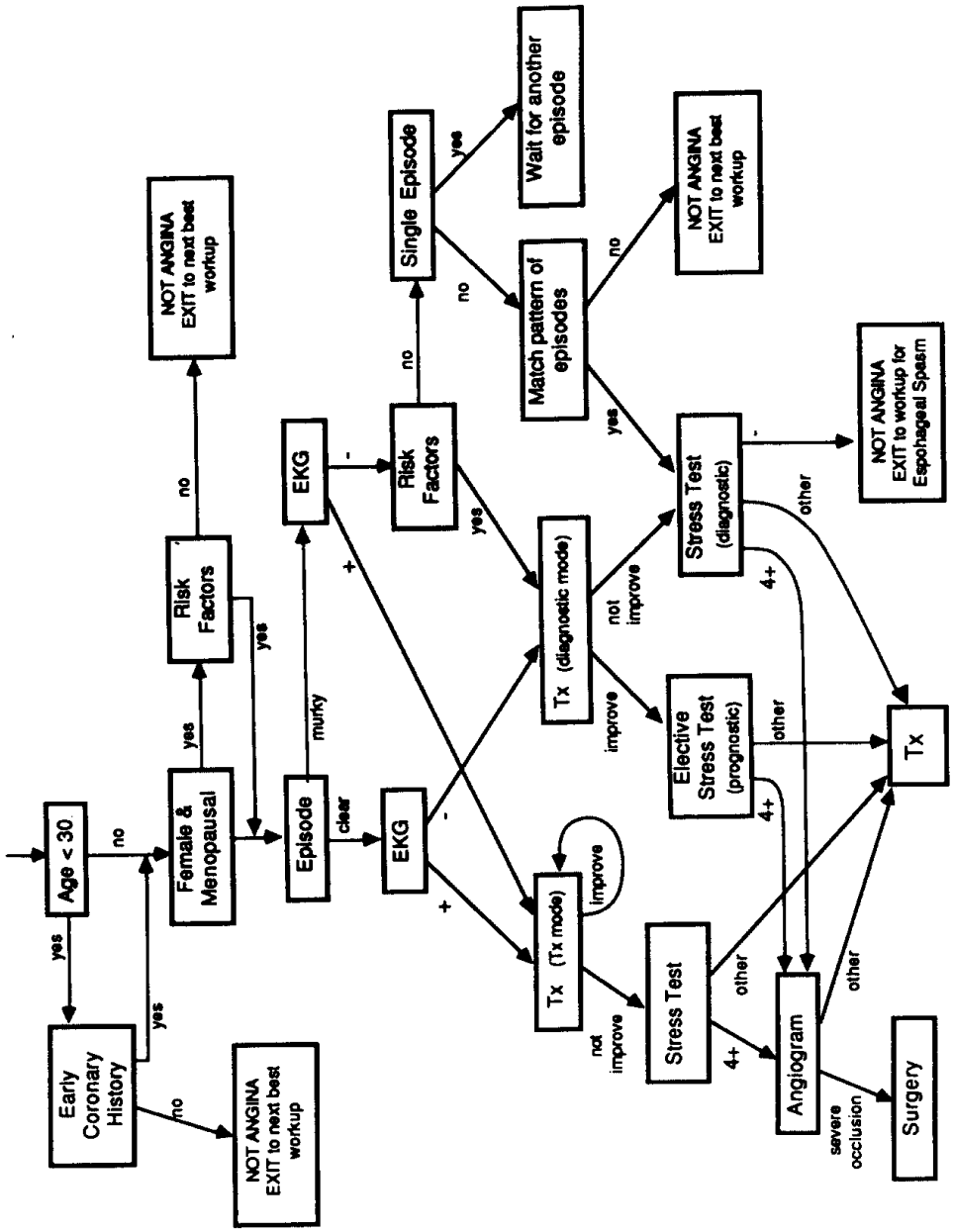

Figure 1. Workup for Angina (Note: Tx refers to nonsurgical treatment)

other dimensions, then problem solving can be guided by the ideal of maximum evidence for minimum cost. For example, here is a strategy for focusing attention on available evidence:

CONTEXT: To minimize cost
CONDITIONS: $Test_1$ and $test_2$ are pertinent, and
$test_1$ is potentially-confirming, and
$test_2$ is potentially-supporting, and
cost($test_1$) $\gg$ cost($test_2$)
ACTIONS: Begin
Do $test_2$
If supporting then do $test_1$,
else do not do $test_1$
End

That is, according to this strategy, given cheap, weak evidence and expensive, strong evidence, one should acquire the weak evidence first and not incur the cost of the strong evidence unless the weak evidence lends support. The rule thus serves to manage the uncertainty associated with the weak evidence. It also uses features of evidence such as cost and reliability to control the acquisition of evidence; for example, it explains why an angiogram—an expensive, risky, and painful test—is done only after a stress test (see Figure 1). We distinguish these functions—managing uncertainty and control—only because uncertainty and control have, with a few exceptions (noted below), been viewed as different topics. In fact, if control decisions are based on features of evidence, then control and managing uncertainty are the same thing. This is the principle that motivates the design of MUM.

Related Work

The close association between control and managing uncertainty has been apparent in the literature on sophisticated control for several years* but is largely absent from the artificial intelligence (AI) literature on reasoning under uncertainty. Three important results have emerged from research on control. First, complex and uncertain problems have to be solved opportunistically and asynchronously—working on subproblems in an order dictated by the availability and quality of evidence (Hayes-Roth and Lesser [7]). Second, as control tends to be accomplished by local decisions about focus of attention, the behavior of complex knowledge systems sometimes lacks global coherence. Coherence can be achieved by planning sequences of actions instead of selecting individual actions by local criteria.† Third, programs are impossible to

* See, for example, the classic paper by Erman, L., Hayes-Roth, F., Lesser, V., and Reddy, D., The Hearsay-II Speech Understanding System: Integrating Knowledge to Resolve Uncertainty, *Computing Surveys* 12(2), 213–253, 1980.

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understand if the factors that affect control decisions are implicit. For example, the focus of attention in Hearsay-II was difficult to follow because it depended on many numerical parameters calculated from data and combined by empirical functions with "tuning" parameters [7]. A better approach is to explicitly state and reason about the implicit factors, called control parameters (Wesley [12]), which the numbers represent (Davis [4], Clancey [2]). If the control parameters are features of evidence and uncertainty, then control strategies can be developed to manage uncertainty.

This last point colors our reading of the AI literature on reasoning under uncertainty. Much of it is concerned with the mathematics of combining evidence, the calculation of degrees of belief in hypotheses. (A representative sample includes Shortliffe and Buchanan [10], Duda, Hart, and Nilsson [5], Zadeh [13], and Shafer [9]. See Cohen and Gruber [3] and Bonissone [1] for literature reviews, including non-numeric approaches to uncertainty; and Szolovits and Pauker [11] for a discussion of uncertainty in medicine.) Degrees of belief can serve as control parameters, but it is necessary to maintain a distinction between combining evidence and control. Otherwise, degrees of belief (and the functions that combine them) have to be "tuned" not only to find the most likely answer but also to focus attention in a reasonable way. Inevitably they become ambiguous summaries of implicit control parameters. For example, MYCIN's certainty factors contained probabilistic and salience information, an indirect result of using them to focus attention (Shortliffe and Buchanan [10]).

Another important reason to maintain the distinction between combining evidence and control is that combining evidence is only a part of the problem of reasoning under uncertainty. Other aspects include formulating decisions, assessing the need for more evidence, planning how to get it, deciding whether it is worth the cost and, if it is not, hedging against residual uncertainty. In MUM we address the problem of combining uncertainty in the context of these other tasks.

AN ARCHITECTURE FOR MANAGING UNCERTAINTY

Managing uncertainty in MUM requires several kinds of knowledge that are discussed in this section. Anticipating the section on control, we find it useful to think of data moving bottom up through Figure 2 as they trigger hypotheses and are requested by MUM's planner.

Types of Knowledge

DATA, EVIDENCE, AND INTERPRETATION FUNCTIONS Evidence is abstracted from data through interpretation functions. All data about a patient are stored in frames that describe personal history, family history, test results, history of

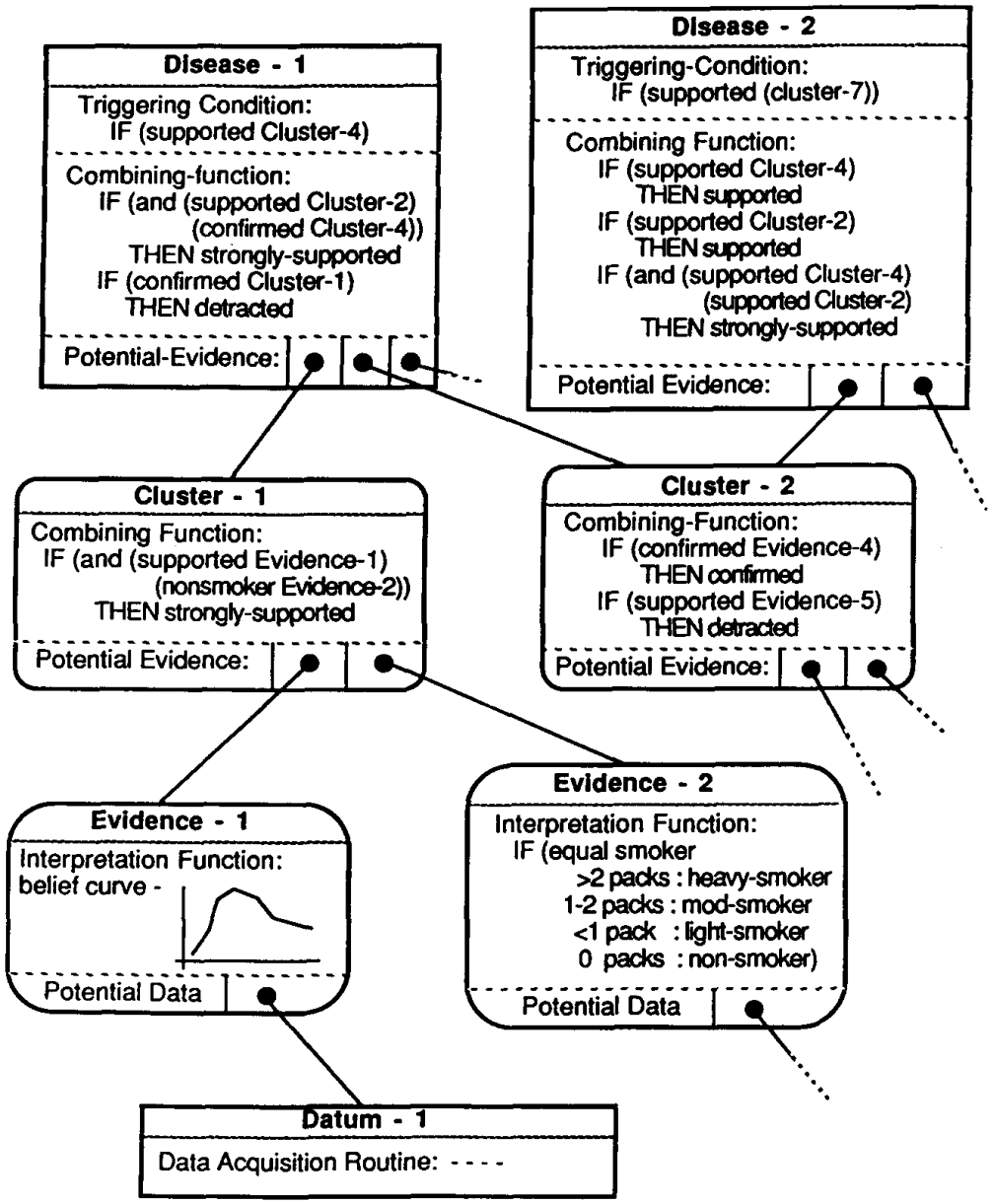

Figure 2. Knowledge Structures in MUM

episodes, and other information. Interpretation functions map data to evidence; for example, information that a patient smokes three packs of cigarettes a day is abstracted to the evidence *heavy smoker* by an interpretation function that maps data about smoking habits to one of the following: nonsmoker, light smoker, moderate smoker, heavy smoker. Interpretation functions are often graphs called belief curves that relate ranges of a continuous data variable to belief in evidence. Figure 3 shows a belief curve relating the duration of chest pain to the evidence classic anginal pain.

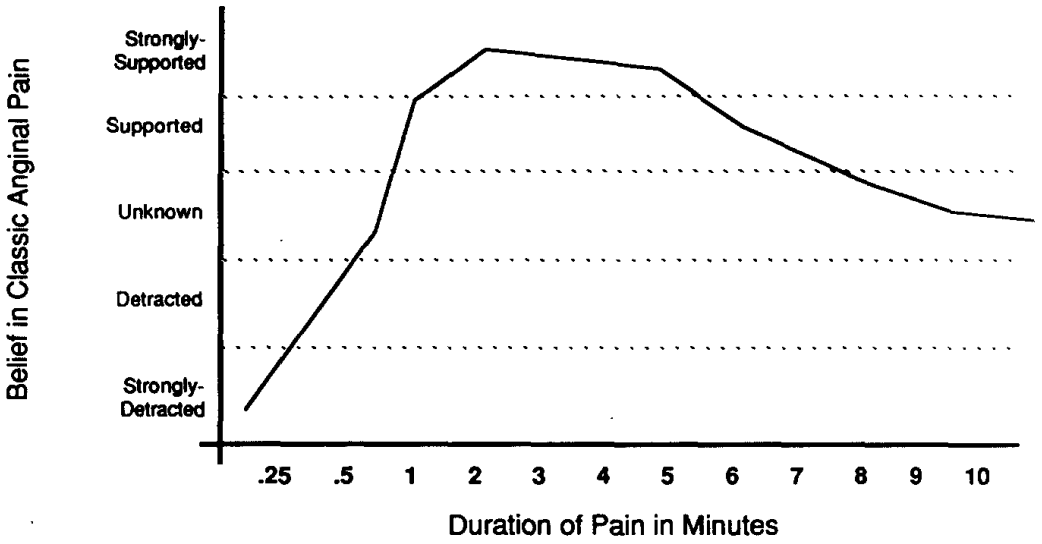

Figure 3. Belief Curve Plotting the Datum "Duration of Pain in Minutes" versus Belief in the Evidence "Classic Anginal Pain"

Belief curves and other interpretation functions are acquired from an expert. They provide the same functionality as Zadeh's fuzzy predicates [13] and generalize Clancey's view of data abstraction as categorical [2].

FEATURES OF EVIDENCE Evidence may be characterized by its cost, reliability, and roles. The cost of evidence reflects monetary cost as well as discomfort and risk to the patient (later versions of MUM will separate these and other determinants of cost). Reliability refers to several factors, including false-positive and miss rates of tests, and also the belief in evidence derived from belief curves (*e.g.*, Is classic anginal pain at least supported by data about the pain duration?).

The most important feature of evidence is the roles it can play with respect to evaluating hypotheses. MUM recognizes five roles, two of which are symmetric pairs.

1. *Potentially confirming and potentially disconfirming.* If evidence plays a potentially confirming role with respect to a hypothesis, then acquiring it might confirm the hypothesis, although not all potentially confirming evidence will in actuality confirm. For example, an EKG confirms the hypothesis of angina only if "positive" (*i.e.*, showing ischemic changes). Once confirmed (or disconfirmed), a hypothesis requires no further evidence, although a diagnostician may continue working to disconfirm other hypotheses, especially if they are dangerous.
2. *Potentially supporting and potentially detracting.* Such evidence is like potentially confirming and potentially disconfirming, but not conclusive. However, combinations of supporting or detracting evidence may be

confirming and disconfirming, respectively (see "Combining Functions," below). The combination referred to as cluster-2 (Figure 2) is potentially supporting with respect to disease-2; cluster-1 is potentially detracting with respect to disease-1.

3. *Trigger.* Evidence plays the triggering role with respect to a hypothesis if its presence focuses attention on the hypothesis or "brings the hypothesis to mind," or, in MUM, adds the hypothesis to a list of potential diagnoses. Cluster-4, if it is supported, triggers disease-1 (Figure 2). This role of evidence is found in virtually all medical expert systems.
4. *Modifying.* Some evidence primarily alters the way diagnosis proceeds. For example, risk factors for coronary artery disease (*e.g.,* hypertension, elevated cholesterol) play a *modifying* role with respect to the hypothesis of angina, as the diagnosis will proceed aggressively if they are present and less aggressively otherwise.

   Note that evidence can play multiple roles with respect to any hypothesis; for example, risk factors are both potentially supporting and modifying with respect to angina; and most triggers are individually or in combination with other evidence at least potentially supporting (*e.g.,* note the roles cluster-4 plays with respect to disease-1 in Figure 2). Also, one piece of evidence can play different roles with respect to several hypotheses (illustrated by the roles cluster-2 plays with respect to disease-1 and disease-2 in Figure 2). Finally, note that some evidence potentially plays two symmetric roles, whereas some is "asymmetric"; for example, a stress test will either support coronary artery disease or detract from it, whereas an EKG supports angina if it is positive and is useless otherwise. That is, EKG plays a potentially supporting role only.
5. *Clusters.* Physicians often see collections of evidence that play particular roles in diagnosis; for example, shortness of breath that comes on suddenly but is unrelated to exercise (or other inciting factors) triggers the diagnosis of pulmonary embolism. Just as evidence has roles with respect to clusters, so clusters have roles with respect to diseases, and these roles need not be supporting; for example, the cluster (patient age < 30 and no family history of coronary events) plays a potentially disconfirming role with respect to all coronary diagnoses of chest pain. Instead of saying that the available evidence is a poor match to coronary diagnoses, we can say the evidence is a good match to a cluster that potentially detracts from or disconfirms coronary diagnoses.

COMBINING FUNCTIONS Every cluster includes a function, specified by the expert, that combines the available evidence for the cluster and returns a value for the cluster-given evidence. The values returned by combining functions are just "realizations" of potential roles of evidence. For example, the value

returned by the combining function of a cluster supported by potentially confirming evidence could be confirmed. The value for a cluster with several pieces of potentially detracting evidence might be disconfirmed or perhaps detracted. (Combining functions are further discussed below.)

DISEASES A disease is technically a cluster. Diseases reside at the top of a hierarchy of clusters (as shown in Figure 2), each of which has its own combining function and specifications of the roles played by the clusters below it.

STRATEGIC KNOWLEDGE We characterize strategic knowledge as heuristics for deciding which triggered disease hypotheses to focus on and how to go about selecting actions to gather evidence pertinent to these hypotheses. These heuristics have the same contingent nature as Davis' meta-rules [4] and control rules in Neomycin (Clancey [2]). We represent strategies as rules that include:

Conditions for selection of the strategy.
A focus policy that guides the choice of a subset of the triggered disease hypotheses to focus on.
Planning criteria that guide the selection of actions to gather evidence and treat diseases currently in the focus.

Examples of focus policies are plausibility (choosing hypotheses on the basis of their degree of support); criticality (focusing on hypotheses that, if true, would require immediate action); and differentiation (focusing on hypotheses that offer alternative explanations for the symptoms). Examples of planning criteria are cost (evidence that is easy to obtain and inexpensive on some cost metric, such as money and time, is preferable); roles (potentially confirming roles are preferred over potentially supporting ones); and diagnosticity (a given result has the potential to increase the belief in one hypothesis and decrease belief in the other, as indicated by belief curves).

Combining Evidence and Propagating Belief

In MUM evidence is combined by local functions, as shown in Figure 2. Typically, knowledge systems require three functions to combine evidence and propagate belief. These are illustrated in the context of two inference rules:

R1: (A AND B)→C

R2: (D AND E)→C

One function calculates the degree of belief (**dob**) in a conjunction from degrees of belief in the conjuncts:

$$\textbf{dob}\ (\text{AND A B}) = F_1(\textbf{dob}\ (A), \textbf{dob}\ (B))$$

The second function calculates the degree of belief in a conclusion from the degree of belief in its premise (computed by F_1) and the "conditional" degree of belief in the conclusion given the premise, often called the degree of belief in the inference rule:

$$\textbf{dob}\ (C_{R1}) = F_2(\textbf{dob}\ (\text{AND A B}), \textbf{dob}\ (C|(\text{AND A B})))$$

The third increases the degree of belief in a conclusion when it is derived by independent inferences:

$$\textbf{dob}\ (C_{R1\&R2}) = F_3(\textbf{dob}\ (C_{R1}), \textbf{dob}\ (C_{R2}))$$

In MUM these three kinds of combining are maintained, but with two important differences. First, there are no global functions corresponding to F_1, F_2, and F_3; all combining is done by functions local to clusters. Second, instead of the usual numeric degrees of belief, MUM has seven levels of belief: *disconfirmed, strongly detracted, detracted, unknown, supported, strongly supported, confirmed.* These are "realizations" of the roles of evidence described earlier.

Combining evidence and propagating belief in MUM is illustrated in Figure 2. Each cluster, including diseases, has its own local combining function, specified by an expert. For example, cluster-1 is strongly supported if evidence-1 is supported and if the data on a patient's smoking habits support evidence that he or she is a nonsmoker. This is a conjunction of evidence of the kind calculated by F_1 above. Another example is found in the combining function for disease-1. If cluster-2 and cluster-4 are both confirmed, then disease-1 is strongly supported. This illustrates the kind of combining for which F_2 above is required: even when the evidence for a disease is itself certain, the conditional belief in the disease given the evidence may not be certain. Disease-2 also contains a conjunctive rule, but the entire combining function illustrates the corroborative situation for which F_3 is needed. In this case, cluster-4 and cluster-2 individually play potentially supporting roles and taken together increase the level of belief in disease-2 to strongly supporting.

Local combining functions have many advantages. Foremost among them is the ease with which an expert can specify precisely how the level of belief in a cluster depends on the levels of belief in the evidence for that cluster. Control of combining evidence is not relinquished to an algorithm but is acquired from the expert as part of his or her expertise. Because local combining functions are specific to clusters, they can be changed independently. And because the values passed between them in MUM are few, it is easy to trace back the derivation of a level of belief and pinpoint a faulty local combining function. The prospect of having to acquire many functions seems daunting, but we have found it easy and intuitive, and much easier to explain than a global numeric method.

Control of Diagnosis in MUM

MUM's basic control strategy involves three components. The *user interface* uses data description frames in the knowledge base to ask questions and create patient data frames for the results. The *matcher* uses the interpretation and combining functions to record the effect that incoming data have on the belief states for clusters and disease frames, and triggers new hypotheses as appropriate. Finally, the *planner* uses strategic control rules to guide the selection of a focus set and the planning process.

BASIC CONTROL The planner follows a basic control loop within which it interprets strategic control rules. It is implemented in a blackboard system with knowledge sources specified in the same syntax as strategic control rules. This facilitates experimental modifications. The design of the blackboard system was influenced by Hayes-Roth [6] and shares the emphasis on explicit solution to the control problem. We first describe the basic control loop, then strategies and their selection.

The basic control loop is initiated with the choice of a strategic phase. All strategic phases but one include a focus policy that directs MUM's attention to a subset of candidate hypotheses. This is followed by the selection of short-term plans to gather evidence and select treatment pertinent to these hypotheses (the rule in the Introduction represents such a plan). Because the effort of developing lengthy plans may well be wasted in a domain permeated with uncertainty, we constrain plans to single actions or sequences of two actions, where the applicability of the second depends on the outcome of the first. Several short-range plans may be generated and executed.

Carrying out plans typically involves invoking the user interface to request some information, updating the status of the diseases with the matcher, and conditional continuation of the plan. When no short-term plans remain, the system iterates the basic control loop to determine if a new strategic phase is appropriate, updates the focus, and generates new short-term plans.

STRATEGIC CONTROL We represent MUM's overall strategy as an ordered set of rule-like strategic phases, shown in Figure 4. Each phase has conditions that activate it. Once activated, a phase controls MUM's focus of attention and the choice of actions pertaining to the hypotheses in this focus.

The phase *Get General Picture* is invoked when the system is started and may also be used if all previously considered hypotheses are ruled out. It has no focus policy because no hypotheses are active when it is invoked. It directs the planner to ask for evidence that plays the potential trigger role for one or more hypotheses, pursuing the lowest-cost evidence first. The cluster *initial consultation* (consisting of age, sex, and primary complaint) meets the criteria

Strategic Phase:	Get General Picture.
Conditions:	No candidate hypotheses.
Focus Policy:	None.
Planning Criteria:	Evidence must play trigger role; prefer low cost on all cost metrics.
Strategic Phase:	Initial Assessment for Triggered Hypotheses.
Conditions:	One or more hypotheses are triggered.
Focus Policy:	Focus on triggered hypotheses.
Planning Criteria:	Must be low on all cost metrics; prefer stronger roles.
Strategic Phase:	Deal With Critical Possibilities.
Conditions:	There are critical hypotheses that have not been confirmed, disconfirmed, or strongly detracted; if they are detracted, no other hypothesis is confirmed.
Focus Policy:	Criticality.
Planning Criteria:	Rule out if possible, else gather support. Utility of evidence. Low cost first; as needed, let discomfort and monetary cost increase.
Strategic Phase:	Discriminate Strongest Hypotheses.
Conditions:	More than one hypothesis is supported.
Focus Policy:	Plausibility.
Planning Criteria:	Diagnosticity. Low cost first. Utility of evidence. Substitute high-cost confirmation for one hypothesis with lower-cost disconfirmation for the other.

Figure 4. Four Strategic Phases in MUM's Diagnosis

of potentially triggering many hypotheses and costing little. The initial consultation usually triggers some hypotheses, which result in selection of a new strategic phase. If no hypotheses are triggered, the planner asks for potential triggers of higher cost.

The *Initial Assessment for Triggered Hypotheses* phase is invoked when new hypotheses are triggered. Because the conditions of the other strategic phases depend somewhat on the level of belief in candidate hypotheses, this phase gathers preliminary evidence for the hypotheses. The focus is on the triggered hypotheses, so only evidence playing some role relative to these hypotheses is considered by the planner. This phase directs the planner to gather low-cost evidence for the hypotheses. For example, MUM asks about aspects of the patient's episode (the primary complaint event) that bear on the triggered hypothesis, and about risk factors.

As soon as the easy questions for triggered hypotheses have been asked, MUM decides between the next two phases on the basis of its belief in the

hypotheses and whether any of the hypotheses are critical, that is, require immediate treatment if supported. Critical hypotheses are dealt with first.

The *Deal With Critical Hypotheses* phase places all candidate critical hypotheses in MUM's focus. The short-range planner is then directed to attempt to rule out these hypotheses. It begins with potentially disconfirming or potentially detracting evidence. If it fails to find any, then it looks for potentially supporting evidence. It will not seek evidence that plays a lesser potential role than evidence it already has. For example, it will not seek potentially supporting evidence for a hypothesis that is already strongly supported, but rather focuses on potentially confirming evidence. The planner will focus on low-cost evidence first, but it is not prohibited from pursuing high-cost evidence as it was in the previous phase.

If the focus of attention is not captured by critical hypotheses, it is dictated by plausibility. The strategic phase *Discriminate Strongest Hypotheses* discriminates among competing alternatives with as little cost to the patient as possible. As before, the potential roles of evidence are used to decide whether it is worth acquiring.

Currently, MUM stops work when a hypothesis is confirmed and no critical hypotheses remain in its focus. We are implementing the next strategic phases, prognosis and treatment. Both provide evidence of diagnostic significance; for example, MUM may begin treatment for angina if it is strongly supported rather than incur the cost of absolute confirmation. If the treatment relieves the symptoms, then it is additional evidence for the diagnosis. If not, it is evidence that detracts from the diagnosis and may support others. Because treatment provides evidence, we represent treatments as clusters, exactly the same way as we represent tests such as angiography.

CONCLUSIONS

MUM manages uncertainty by reasoning about evidence and its current state of belief in hypotheses. Its goal is to generate appropriate workups for chest and abdominal pain. The emphasis in MUM is on asking the right questions in the right order without superfluous questions. MUM's control knowledge is not yet sophisticated enough to satisfy all these criteria. It asks questions in a reasonable order, but it sometimes focuses on the wrong disease. Because MUM is a nascent system, this does not yet concern us. We believe the system is successful in providing a framework for exploring management of uncertainty by sophisticated control—that is, by making control decisions based on the roles, costs, and other characteristics of evidence; the criticality of diseases; and the credibility of diagnoses.

Much work remains to be done. Currently, MUM resembles a programming environment more than a medical expert system. We will be devoting ourselves to building up its clusters and control rules.

Although MUM was designed for medical problems and is discussed in that context, we believe the approach to uncertainty and control it engenders is general to classification problem solvers as well as to other systems responsible for the management of uncertainty. An empty version of MUM called MU is being developed and will be tested in other domains.

References

1. Bonissone, P., Reasoning with uncertainty in expert systems, *Int. J. Man-Machine Stud.* 22(3), 241–250, 1985.

2. Clancey, W., The advantages of abstract control knowledge in expert systems design, *Proceedings of the 3rd National Conference on Artificial Intelligence,* Washington, DC, 74–78, 1983.

3. Cohen, P., and Gruber, T., Reasoning About Uncertainty: A Knowledge Representation Perspective, Pergamon Infotech State-of-the-Art Report, 1985.

4. Davis, R., Interactive transfer of expertise, in *Rule-Based Expert Systems* (B. Buchanan and E. Shortliffe, Eds.) Addison-Wesley, Reading, Mass., 1985.

5. Duda, R. O., Hart, P. E., and Nilsson, N., Subjective Bayesian Methods for Rule-Based Inference Systems, Technical Note 124, AI Center, SRI International, Menlo Park, Calif., 1976.

6. Hayes-Roth, B., A blackboard architecture for control, *AI* 26, 251–321, 1985.

7. Hayes-Roth, F., and Lesser, V., Focus of attention in the Hearsay-II speech understanding systems, *Proceedings of the 5th International Joint Conference on Artificial Intelligence,* Cambridge, Mass., 27–35, 1977.

8. Patel, V. and Groen, G., Knowledge based solution strategies in medical reasoning, *Cog. Sci.* 10, 91–116, 1986.

9. Shafer, G., *A Mathematical Theory of Evidence,* Princeton U.P., Princeton, N. J., 1976.

10. Shortliffe, E., and Buchanan, B., A model of inexact reasoning in medicine, *Math. Biosci.* 23, 351–379, 1975.

11. Szolovits, P., and Pauker, S. G., Categorical and probabilistic reasoning in medical diagnosis, *AI* 11, 115–144, 1978.

12. Wesley, L. P., Reasoning about control: The investigation of an evidential approach, *Proceedings of the 8th International Joint Conference on Artificial Intelligence* (IJCAI-83), Karlsruhe, West Germany, 203–206, 1983.

13. Zadeh, L. A., Fuzzy logic and approximate reasoning, *Synthese* 30, 407–428, 1975.